Suppression and enhancement of the critical current in multiterminal S/N/S mesoscopic structures.

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We analyse the measured critical current I_m in a mesoscopic 4-terminal S/N/S structure. The current through the S/N interface is shown to consist not only of the Josephson component $I_c \sin\varphi$, but also a phase-coherent part $I_{sg}\cos\varphi$ of the subgap current. The current I_m is determined by the both components I_c and I_{sg}, and depends in a nonmonotonic way on the voltage V between superconductors and normal reservoirs reaching a maximum at $V \cong \Delta/e$. The obtained theoretical resultas are in qualitative agreement with recent experimental data.

Recent achievements in nanotechnology have revived interest in the study of nonequilibrium and phase-coherent phenomena in superconductor-normal metal (S/N) structures. One of the most remarkable, discovered recently [1], was the observation of the sign reversal of the Josephson critical current I_c (the so-called π-junction) in a multi-terminal mesoscopic Nb/Au/Nb structure under nonequilibrium conditions. By passing an additional current through the N layer or, in another words, by applying a voltage V to the normal reservoirs (see Fig.1) with respect to the superconductors, one can create a nonequlibrium electron-hole distribution, or at least one can shift this distribution with respect to the electron-hole distribution in the superconductors. Under this condition, the critical current I_c decreases with V and changes sign at a certain value of the applied voltage V. This effect was predicted first in Ref. [2] where a ballistic 3-terminal structure was considered (for more details, see also Refs. [3,4]). In diffusive 4-terminal S/N/S structures, the sign-reversal effect has been considered in Refs. [5–7] (see also [8,9]). The sign-reversal effect and switching of the π-junction into a state where $\varphi = \pi$ has much in common with an instability of an uniform superconductor with a nonequilibrium distribution function [10,11].

In multi-terminal S/N/S structures one can observe not only the sign reversal effect, but also a number of other interesting phenomena. For example, the conductance of a normal wire between N reservoirs oscillates with varying phase difference φ (see review articles [12,13]). In addition, as shown in Refs. [5,14], the measured critical current I_m depends on the geometry of a particular structure and instead of decreasing may also increase with increasing voltage V. In particular one can observe Josephson-like effects (plateau on the $I_3(V_S)$ curve, oscillations of the measured critical current I_m in a magnetic field etc) even if the Josephson coupling between superconductors under equilibrium conditions is negligable. The reason for these effects is that the current I_m in a multi-terminal S/N/S structure is determined not only by the Josephson component $I_c \sin\varphi$, but also by the phase-dependent subgap current $I_{sg}\cos\varphi$ through the S/N interface. Therefore even in the case of a small I_c, the current I_m can be altered by varying the phase ϕ. An increase of the critical current was observed in the recent paper [15] where a mesoscopic three-terminal S/N/S structure was studied. The authors used a third superconductor as a reservoir the electric potential of which was shifted with respect to the other two superconductors by the voltage V. The measured critical current reached its maximal value when the magnitude of V was comparable with Δ. At some, not too low temperatures T the measured critical current I_m exceeds its magnitude in the equilibrium state: $I_m(V) > I_m(0)$. In the present paper we show that the enhancement of the supercurrent observed in Ref. [15] is most likely caused by the mechanism mentioned above. In Refs. [5,14] the model case of gapless superconductors was considered where there is no singularity in the density-of states in superconductors at $\epsilon = \Delta$. Here we will consider the case of ordinary superconductors with an energy gap Δ and show that the enhancement of the critical current reaches a maximum for V of order Δ. The voltage dependence $I_m\ (V)$ calculated for different temperatures is in qualitative agreement with the experimental data.

We consider the structure shown in Fig.1 which differs from the structure studied experimentally. However in our opinion, this difference is not essential and allows us to give at least qualitative explanation for the phenomena observed in Ref. [15]. First, we assume for simplicity that the structure under consideration is symmetrical, i.e. it has four terminals and not three as in the experiment. Secondly, we consider normal reservoirs in order to avoid complications which would arise in case of superconducting reservoirs (ac Josephson effects when the finite voltage is applied to the S reservoir). We also assume for simplicity that the contacts between the N wire and N reservoirs are good (the resistance of the N wire/N reservoir interface is much smaller than the resistance of the N wire), whereas the S/N interface resistance is finite (larger or less than the resistance of the N wire). We will study the diffusive case which corresponds to the experiment [15].

In order to find the dependence of the effective critical current $I_m(V)$ (the definition of $I_m(V)$ will be given later), we need to determine two distribution function f_+ and f_-. Both these functions are isotropic in space. The function f_+ is related to a symmetrical part of the distribution function in the electron-hole space:$f_+(\epsilon) = 1-(n_\uparrow(\epsilon)+p_\downarrow(\epsilon)) = 1-(n_\downarrow(\epsilon)+p_\uparrow(\epsilon))$, here $p_\downarrow(\epsilon) = 1-n_\downarrow(-\epsilon)$ is the hole distribution function. It determines the critical current I_c. The function $f\ _-$ describes the electron-hole imbalance and determines the electric potential and current: $f(\epsilon) = -(n_\uparrow(\epsilon)-p_\downarrow(\epsilon)) = -(n_\downarrow(\epsilon)-p_\uparrow(\epsilon))$. Equations for f_+ and f_- are obtained from an equation for the matrix Keldysh function $\hat{G}$ (see, for example [5,13] For the structure shown in Fig.1 they can be written in the form

$$L\partial_x(M_-\partial_x f_-(x)+J_s f_+-J_{an}\partial_x f_+(x)) = r[A_-\delta(x-L_1)+\bar{A_-}\ \delta(x+L_1)]. \tag{1}$$

$$L\partial_x(M_+\partial_x f_+(x)+J_s f_-+J_{an}\partial_x f_-(x)) = r[A_+\delta(x-L_1)+\bar{A_+}\ \delta(x+L_1)] \tag{2}$$

Here all the coefficients are expressed through the retarded (advanced) Green's functions $\overset{\wedge}{G^R}= G^R\hat{\sigma}_z+\overset{\wedge}{F^R}$ and are equal to $M_\pm = (1-G^RG^A \mp (\overset{\wedge}{F^R}\overset{\wedge}{F^A})_1)/2$;

$J_{an} =(\overset{\wedge}{F^R}\overset{\wedge}{F^A})_z/2$, $J_s = (1/2)(\overset{\wedge}{F^R}\partial_x\overset{\wedge}{F^R} - \overset{\wedge}{F^A}\partial_x\overset{\wedge}{F^A})_z$, $A_- = (\nu\nu_S+g_{1+})f_- - (g_{z-}f_{eq}+g_{z+}f_+)$; $A_+ = (\nu\nu + g_{1-})(f_+-f_{eq}) - g_{z-}f_-$; $g_{1\pm} = (1/4)[(\overset{\wedge}{F^R}\pm\overset{\wedge}{F^A})(\overset{\wedge}{F^R_S}\pm\overset{\wedge}{F^A_S})]_1$; $g_{z\pm} = (1/4)[(\overset{\wedge}{F^R}\mp\overset{\wedge}{F^A})(\overset{\wedge}{F^R_S}\pm\overset{\wedge}{F^A_S})]_z$;

The coefficient $r = R/R_b$, $R = \rho L/d$ is the resistance of the N film per unit length in the z-direction,ρ is the specific resistivity of the N film, d is the thickness of the N film, R_b is the S/N interface resistance; the functions $\bar{A_-}$ and $\bar{A}_+$ coincide with A_- and A_+ if we make a substitution $\varphi \to -\varphi$. We introduced above the following notations $(\overset{\wedge}{F^R}\overset{\wedge}{F^A})_1 = Tr(\overset{\wedge}{F^R}\overset{\wedge}{F^A})/2$, $(\overset{\wedge}{F^R}\overset{\wedge}{F^A})_z = Tr(\hat{\sigma}_z\ \overset{\wedge}{F^R}\overset{\wedge}{F^A})/2$ etc.; ν and ν_S are the density-of states in the N film at $x = L_1$ and in the superconductors. The boundary conditions for f_+ and f_- are: $f_+\ (L) = F_{V+}$ and $f_-(L) = F_{V-}$; the functions $F_{V\pm}$ are the corresponding distribution functions in the normal reservoirs: $F_{V\pm} = [\tanh((\epsilon+eV)\beta) \pm \tanh((\epsilon-eV)\beta)]/2$. We set the electrical potential at the superconductors equal to zero and assumed that the width of the S/N interfaces w is small compared to $L_{1,2}$.

Eq.(1) describes the conservation of the electric current (at a given energy). The term in the brackets on the left is the total partial current in the N wire, consisting of the quasiparticle current (the first term), the supercurrent in the interval $(-L_1, L_1)$ (the second term) and a "nonequilibrium supercurrent" (the third term). The coefficient M is a quantity which is proportional to the diffusion coefficient renormalised due to proximity effect. The right hand side is the partial current through the S/N interface; the term $(\nu\nu_S+g_{1+})f_-$ is the quasiparticle current above $(\nu\nu_S f_-)$ and below $(g_{1+}f_-)$ the gap. The term $(g_{z-}f_{eq}+g_{z+}f_+)$ is the Josephson current in nonequilibrium conditions. Eq.(2) describes the conservation of the energy flux (at a given energy). The coefficient A_+ is zero below the gap (complete Andreev reflection) as the difference $(F^R_S - F^A_S)$ equals zero at $\epsilon < \Delta$.

The solutions of Eqs.(1)-(2) can be found exactly and expressed in terms of the retarded (advanced) Green's functions which obey the Usadel equation. First we note that the expressions in brackets in the left hand side of Eqs.(1)-(2) in the regions $(0, L_1)$ and (L_1, L) are equal to the constants of integration $C_{1,2\pm}$. The constants $C_{1,2-}$ relate to partial currents $J_{1,2}$ $(C_{1,2-} = eJ_{1,2}\rho/d)$. The partial currents $J_{1,2}$ are the currents per unit energy and connected with the electrical currents $I_{1,2}$ via the relation

$$I_{1,2} = \int_0^\infty d\epsilon J_{1,2}(\epsilon) \tag{3}$$

Our aim is to find the current I_3 and express it in terms of the control current I_2 (or voltage V) and the phase difference φ. We note that the distribution functions $f_\pm(x)$ are constants in the region $x \in (0, L_1)$ and vary in the region $x \in (L_1, L)$ reaching $F_{V\pm}$ at $x = L$. Dropping details of calculations, we present final results for limiting cases.

a) Large interface resistance: $r << 1$.

One can show that in this case $f_+(0) \cong (F_{V+}+f_{eq}(r_2\nu\nu_s))/(1+r_2\nu\nu_s)$ and $f_-(0) \cong F_{V-}/(1+r_2\nu\nu_s)$, where $r_2 = r(L_2/L)$. The current I_3 through the S/N interface consists of three terms

$$I_3(V) = I_o(V) - I_c(V)\sin\varphi + I_{sg}(V)\cos\varphi \tag{4}$$

Two of them $(I_o, I_{sg}\cos\varphi)$ are the quasiparticle currents and one $(I_c\sin\varphi)$ is the Josephson current. This expression shows that at a given control voltage V and zero voltage difference between the superconductors (φ is constant in

time) the current I_3 may vary with changing φ in the limits:$\mid I_3(V) - I_o(V) \mid \leq I_m(V)$. This means a plateau on the $V_S(I_3)$ characteristics (see [5,14]); here $V_S = (\hbar/2e)\partial_t\varphi$ is the voltage difference between superconductors. We can write the phase-dependent part of I_3 in the form $I_{3\varphi} = I_m \sin(\varphi + \alpha)$, where $I_m = \sqrt{I_c^2 + I_{sg}^2}$ is the measured critical current, $\cos\alpha = -I_c/I_m$. In the considered limit of high interface resistance, we have for I_c and I_{sg}

$I_c(eR_b) = -\int_0^\infty d\epsilon\{\mathrm{Im}(F_S(F_y - F_x))f_o(0) + \mathrm{Re}\, F_S\, \mathrm{Im}(F_y - F_x)f(0)\}$

$I_{sg}(eR_b) = \int_0^\infty d\epsilon g_{sg} f(0) = \int_0^\infty d\epsilon\, \mathrm{Im}\, F_S\, \mathrm{Im}(F_y - F_x)f(0);$

Here $\theta = k_\epsilon L, \theta_2 = k_\epsilon L_2, k_\epsilon = \sqrt{(-2i\epsilon + \gamma)/D}$, γ and D is the damping rate and diffusion coefficient in the N film, $g_{sg} = g_{1+}$ is the normalised subgap conductance (see the expression for A). The functions F_y, F_x are the components of the retarded Green's function in the N film: $\hat{F}{}^R = F_x i\hat{\sigma}_x + F_y i\hat{\sigma}_y$, and $F_S = \Delta/\sqrt{(\epsilon + i\Gamma)^2 - \Delta^2}$ is the amplitude of the retarded Green's function in the superconductors. If we linearise the Usadel equation, we obtain $F_y - F_x = 2F_S \sinh^2\theta_2/(\theta \sinh 2\theta)$. We note that the numerical solution of the Usadel equation shows that the linearised solution is a good approximation even if $r \cong 1$ (at $r = 1$ the difference between the exact and the linearized solutions at the characteristic energy $\epsilon = \epsilon_L = D/L^2$ is less than 5 percent). In Fig.2 we plot the V dependence of I_c, I_{sg} and I_m where we see that the real critical current I_c decreases and changes sign with increasing V, whereas the measured critical current I_m first decreases and then increases. Its maximum may exceed $I_c(0)$. The reason for such a behaviour of I_m is the third term on the right side in Eq.(5) which describes a contribution of the phase-dependent part of the subgap quasiparticle current I_{sg} through the S/N interface to the current I_3. The current I_{sg} is zero at $V = 0$ and increases with V; this current leads to a low [16] and high [17] temperature peak in the conductance. Its phase dependence was measured in Ref. [18] and discussed in many papers (see review articles [12,13]). One can see from Fig.2 that due to the current I_{sg} the measured critical current I_m remains finite when $I_c(V)$ turns to zero.

Fig.3 shows the dependence of the measured critical current I_m on the control voltage V for different temperatures. Our results qualitatively agree with the experimental data of Ref. [15]; that is, the current I_m reaches a maximum at $V \cong \Delta/e$ and this maximum exceeds the equilibrium value of $I_c = I_c(0)$ when the temperature is not too low. One can see, in agreement with the experimental results of Ref. [15], the maximal value of I_m depends on the temperature much weaker than I_c. Although it is difficult to carry out a quantitative comparision between theory and experiment because in the experiment the width w and the interface resistance R_b were comparable with $L_{1,2}$ and R respectively, and a superconducting reservoir was used instead of a normal one (therefore, strictly speaking, one must take into account ac Josephson effects).

An important point to note is that our results do not mean that the sign reversal of the real critical current I_c can not be identified directly. Consider for example a fork-shape circuit; this means that two vertical superconducting leads in Fig.1 are attached to a T-shape (inverted) superconducting lead. Analysing the stability of the state with negative I_c, one can easily show that the state with $\varphi = 0$ is unstable with respect to fluctuations of φ and the system switches to a state with a circulating current. Indeed, taking into account the fluctuating voltage at the superconductor $V_S = \hbar\partial_t\varphi/2e$, we replace V in Eq.(4) by $V - V_S$. We then write down the equation for the current $\overline{I_3}$ in the lead attached to the left superconductor; this equation coincides with Eq.(4) if φ is replaced by $-\varphi$. Subtracting these equations for I_3 and $\overline{I_3}$, we arrive at the equation for a circulating current $I_{cir} = -(I_3 - \overline{I_3})/2$:

$$I_{cir} = I_c(V)\sin\varphi + V_S(R_0 + R_{sg}\cos\varphi). \tag{5}$$

where $R_0 = \partial I_o/\partial V$ and $R_{sg} = \partial I_{sg}/\partial V$. Fluctuations of I_{cir} lead to a magnetic flux $\Phi = I_{cir}L/c$ in the loop which is related to φ: $\Phi = \Phi_o\varphi$, here Φ_o is the magnetic flux quantum and we assumed the absence of flux in the ground state. We find readily from Eq.(5) that the state with $\varphi = 0$ is unstable if $I_c(V) < 0$ and $\mid I_c(V) \mid > c\Phi_o/L$, where L is the loop inductance [19].

b) Small interface resistance.

One can show that in this case the function $f_-(0)$ is zero in the main approximation with respect to the parameter $(r\theta)^{-1}$ (this means that the condition $r^2 >> \Delta/\epsilon_L$ should be satisfied; here $\epsilon_L = D/L^2$ is the Thouless energy). The function f_+ , which determines the Josephson current, in the main approximation is equal to F_{V+} at $\mid \epsilon \mid < \Delta$ and to f_{eq} at $\mid \epsilon \mid > \Delta$. Therefore the dependence $I_c(V)$ is similar to that found numerically in Ref. [6] for another geometry (for small interface resistance); that is, the critical current $I_c(V)$ changes sign with increasing V at V of the order of the Thouless energy. As to the current I_2, it does not depend on the phase difference in the main approximation. Indeed, in order to find I_2 we need to solve the Usadel equation in the region $x \in (L_1, L)$ with boundary condition which is reduced to $\hat{G}{}^R = \hat{G}{}^R_S$. Making the gauge transformation $\hat{G}{}^R_S \Longrightarrow \hat{S}\, \hat{G}{}^R_S\, \hat{S}{}^+$, we can exclude the phase (here $\hat{S} = \cos(\varphi/2) + i\hat{\sigma}_z \sin(\varphi/2))$. Therefore in the main approximation the third term in Eq.(4) is zero.

In conclusion, we have studied the dependence of the measured critical current I_m on the voltage V between normal reservoirs and superconductors in a 4-terminal S/N mesoscopic structure. The current I_m is shown to decrease with increasing V, then to increase reaching a maximum at $V \cong \Delta/e$. Our results qualitatively agree with experimental data obtained in the recent paper [15].

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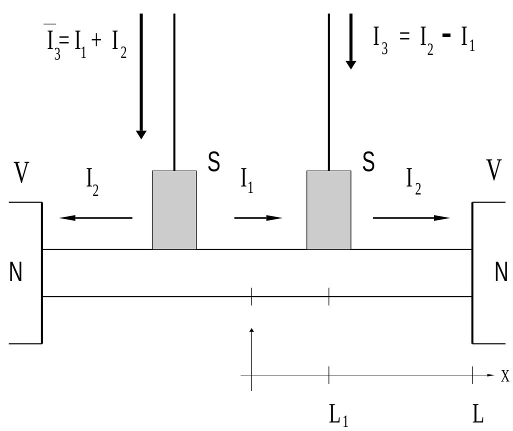

FIG. 1. Schematic view of the 4-terminal S/N/S structure under consideration. The electric potential of the superconductors is zero.

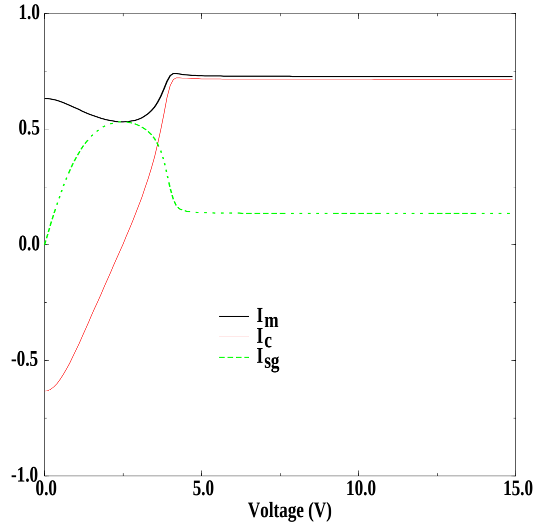

FIG. 2. The measured (I_m) and real (I_c) critical currents vs the control voltage V. The amplitude of the phase-dependent part (I_{sg}) of the subgap current is shown by the dashed line. The currents and voltage are measured in units $\epsilon_L R/eR_b^2$ and ϵ_L/e respectively ($\epsilon_L = \hbar D/L^2$ is the Thouless energy). The parameters are: $\Delta = 4\epsilon_L$,$T = \epsilon_L/4$,$L_1/L = 0.3, r = 0.3$.

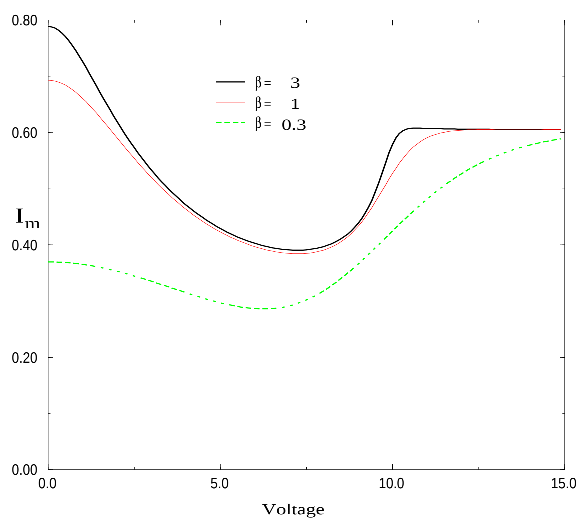

FIG. 3. The measured critical current (I_m) vs V for different temperatures: $\beta = \epsilon_L/2T$. The parameters are: $\Delta = 10\epsilon_L, L_1/L = 0.3, r = 0.3$.